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Tests of Mach's Principle with a Mechanical Oscillator*

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Abstract

James F. Woodward has made a prediction, based on Sciama's formulation of Mach's Principle in the framework of general relativity, that in the presence of energy flow the inertial mass of an object may undergo sizable variations, changing as the 2nd time derivative of the energy. We describe an attempt to observe the predicted effect for a charging capacitor, using a technique that does not require a reactionless force or any local violation of Newton's 3rd law of motion. We attempt to observe the effect of the mass variation on a driven harmonic oscillator with the charging capacitor as the oscillating mass. Positive and negative phase shifts in the oscillator motion with respect to the driving force are predicted to result from appropriately programmed inertial mass variations. The phase shift is constant, so that data may be accumulated over a very large number of oscillation cycles to insure high precision in the phase shift determination. We report on the predicted effect and the design and implementation of the measurement apparatus. At this time, however, we will *not* report on observations of the presence or absence of the Woodward effect.

Introduction

This is a status report on a new experiment to test a prediction based on general relativity and Mach's Principle, which has been supported by the Breakthrough Propulsion Program of NASA.

Einstein's Principle of Equivalence, a cornerstone of general relativity, asserts the exact universal identity of inertial mass and gravitational mass. However, the origins of inertia and its connection to gravitational mass remain obscure. Mach's Principle, the idea that inertia originates in the gravitational interaction of massive objects with the distant matter of the universe, is an attempt to unify gravitational and inertial mass, but it is not a part of general relativity. Dennis Sciama [1,2] attempted to improve this situation by showing that, for sufficiently symmetric and homogeneous universes, the gravitational interaction of massive objects with distant matter leads to an acceleration-dependent force, i.e., inertia.

James F. Woodward [3,4] extended Sciama's calculations by introducing energy flow (e.g., the energy flowing to a charging capacitor) into the gravitating system. He demonstrated that the equations acquire extra transient contributions in Sciama's inertia term that are proportional to 1/G (Newton's gravitational constant) and therefore are quite large. The implications of this work are: (a) that it may be possible to modify inertia, and (b) that it may be possible to demonstrate the validity of Mach's Principle with a tabletop experiment.

Woodward and his students [4-7] have attempted to observe the predicted inertia-variation effect by accelerating a mass-varying object so that it produces a reactionless force. To illustrate this, assume that an inertia-varying test mass is accelerated to the right when it has low inertia and to the left when it has high inertia. In this circumstance, it is argued, the reaction forces of the two accelerations are unequal and one might expect the net reactionless force to "row" the system to the right. Woodward's group reports [7] using a sensitive torsion balance to observed small reactionless forces at magnitudes that are near the limits of their sensitivity and about five orders of

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magnitude smaller than the predicted effect (see the calculations below.)

Unfortunately, this scheme for observing the predicted inertia variation appears to be at odds with the relativistically invariant form of Newton's 2nd law of motion:

$$\vec{F} = d\vec{p}/dt = m\, d\vec{v}/dt + \vec{v}\, dm/dt \qquad (1)$$

Since the inertial mass **m** of the test body is expected to vary with time, the last term of Eqn. (1) cannot be ignored. It is not surprising, in view of Newton's 3rd law of motion, that for any closed cycle of acceleration and variation of the inertial mass around a central value, the force contribution from the **v dm/dt** term is found to precisely cancel the supposed "reactionless force" arising from the **m dv/dt** term, leading to a net force of zero for the overall system.

From this simple calculation, it appears that reactionless force searches are ***not*** good tests of the proposed effect. There remains the question of whether the Woodward inertia variation is indeed present in a system with energy flow. We have found, as will be described below, that a mechanical oscillator, driven at resonance, with its mass programmed to vary at the drive frequency, shows sensitive variations in drive-to-response phase and amplitude, depending on the relative phase between the mass variation and the oscillator drive.

Theory

Woodward has shown [5] that the relativistically invariant wave equation, in the simplest approximation and expressed as a function of an overall scalar gravitational potential ϕ, has the form:

$$\nabla.\phi - (1/c^2)(\partial^2\phi/\partial t^2) = \quad \phi = 4\pi G\rho_o + (\phi/\rho_o c^2)(\partial^2\rho_o/\partial t^2) - (\phi/\rho_o c^2)^2(\partial\rho_o/\partial t)^2 \qquad (2)$$

where G is Newton's gravitational constant, ρ_o the rest mass density, and c the speed of light. This field equation is obtained only if one assumes, as suggested by Mach's Principle, that the local energy density of matter is equal to the matter density times ϕ. Since Mach's Principle demands that $\phi = c^2$ when measured locally, this constraint is equivalent to asserting that $E = mc^2$. Additional terms would be present in this equation were it not for the fact that, as a consequence of Mach's principle in this approximation, $\phi = c^2$.

In writing Eqn. (2), Woodward neglects a term of the form $c^{-4}(\partial\phi/\partial t)^2$ because it is always small, given its c^{-4} coefficient that is not compensated for by any factor of ϕ in the numerator. Combining the last three terms of Eqn. (2) into an effective mass density $\rho(t)$ and solving for this quantity gives the time-dependent effective mass density as:

$$\rho(t) \approx \rho_o + (1/4\pi G)[(\phi/\rho_o c^2)(\partial^2\rho_o/\partial t^2)] - (1/4\pi G)[(\phi/\rho_o c^2)^2(\partial\rho_o/\partial t)^2]. \qquad (3)$$

The second term in Eqn. (3) has the form $(1/4\pi G)[(\phi/\rho_o c^2)(\partial^2\rho_o/\partial t^2)]$. This time-dependent fluctuation in the inertial mass can be both positive and negative when ρ_o undergoes periodic time variations, e.g., when a varying flow of mass-energy is present. This is the inertia-varying term of interest.

In the present work we will ignore the last time-dependent or "wormhole" term, which has the form $-(1/4\pi G)[(\phi/\rho_o c^2)^2(\partial\rho_o/\partial t)^2]$. This mass term is always negative or zero and for sinusoidal variations is about 0.1% or less of the other terms.

If a capacitance **C** is driven by a voltage source with time dependent potential $V(t) = V_0 Sin(\omega t)$, then the energy in the capacitor, assuming dissipative and inductive effects can be neglected, is $U(t) = ½C\, V(t)^2 = ½C\, V_0^2 Sin^2(wt)$. The second time derivative of this stored energy divided by c^2 (to convert it to a mass) is $d^2U/dt^2 = C\, V_0^2\, \omega^2\, Cos(2\omega t)/c^2$. This is the $(\partial^2\rho_o/\partial t^2)$ factor in Woodward's Eqn. (3). The corresponding time dependent variation in inertial mass, assuming that $\phi = c^2$, is then:

$$dm(t) = 1/(4\pi\rho_o G\, c^2)\, C\, V_0^2\, \omega^2\, Cos(2\omega t). \qquad (4)$$

We will use this form for the variation in inertial mass in the analysis that follows.

To give this prediction a scale, let us assume that c=2.998×10^8 m/s, G=6.672×10^{-11} m^3/kg s^2, ρ_0=2,000 kg/m^3, C=9.3×10^{-9} F, V_0=2,000 V, and ω=2π×1,000 Hz. With these values, we find that:

$$dm(t) = 9.7\ mg \times Cos(2\pi \times 2{,}000\ Hz \times t). \qquad (5)$$

In other words, under these conditions, which should be realizable in the experiment described here, the inertial mass of the capacitance is predicted to vary by about ±10 milligrams at *twice* the capacitor charging frequency, or 2,000 Hz. If the mass of the capacitor and its holder were about 1 g, this would represent a mass variation of about ±1%. Such a mass variation would have large observable consequences. However, we note that Woodward [4] has made arguments involving mobile charges to explain why the actual variation in the inertial mass may be orders of magnitude smaller than that predicted by simple calculations and more consistent with his reported observation of very small reactionless forces.

It is also of interest to consider the maximum current flow that is necessary to charge the capacitor in the manner assumed above. The charge on the capacitor is $\mathbf{q=C\ V(t)}$, so the charging current is $\mathbf{i(t) = C\ dV/dt = C\ V_0\ \omega Cos(\omega t) = 116\ mA \times Cos(\omega t)}$ for the specified conditions. It turns out that a high voltage power supply/amplifier capable of delivering an audio-frequency peak current of a few hundred milliamps at a few kilovolts is very expensive (~$14,000) and represent the most costly component required for the present test of the Woodward effect.

A Driven Mass-Varying Oscillator

We test for the presence of the Woodward effect by using the capacitor as the mass in a system that forms a driven mechanical mass-and-spring oscillator with an undriven resonant frequency of $\boldsymbol{\omega_0}$. Such an oscillator is shown schematically in Fig. 1

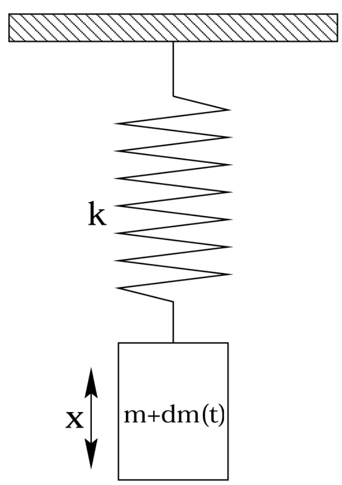

Figure 1. Schematic mass-and-spring mechanical oscillator with time varying mass **m+dm(t)** and restoring-force spring constant **k**. The system is assumed to have a dissipative damping force of **–b dx/dt**.

The oscillator is driven at its resonant frequency $\boldsymbol{\omega_d=(\omega_0^2 - b/2m)^{1/2}}$ with a voice coil actuator and audio amplifier. At the same time, we charge the capacitor sinusoidally, using approximately the parameters specified above, at a frequency of $\boldsymbol{\omega_d/2}$ so that, in the presence of the Woodward effect, the capacitor's inertial mass should vary at frequency $\boldsymbol{\omega_d}$.

The inhomogeneous non-linear differential equation describing such a system is:

$$\begin{aligned} F_d Cos[\omega_d t] = kx(t) + [b + \mu'(t)]\,x'(t) \\ + [m + \mu(t)]x''(t), \end{aligned} \qquad (6)$$

where **x(t)** is the motion of the capacitor, $\mathbf{F_d}$ is the magnitude of the driving force, $\boldsymbol{\omega_d}$ is the angular frequency of the driving force, **k** is the Hooke's law restoring force constant, **b** is the damping constant representing dissipative forces in the system, **m** is the average mass of the capacitor and associated structure, and $\boldsymbol{\mu(t)}$ is the time-dependent mass variation due to the Woodward effect. Note that the $\boldsymbol{\mu'(t)x'(t)}$ term in Eqn. (6) arises from the **v dm/dt** term in Eqn. (1).

We can replace the spring constant **k** with $\mathbf{m\ \omega_0^2}$ and replace $\boldsymbol{\mu(t)}$ with $\boldsymbol{\mu_0 Cos(\omega_d t + \phi_m)}$, which assumes that we have arranged the mass variation to be at the same frequency as the driving force but shifted in phase by $\boldsymbol{\phi_m}$. With these substitutions, Eqn. (6) becomes:

$$\begin{aligned} F_d Cos(\omega_d t) &= m\omega_0{}^2 x(t) \\ &+ [b - \mu_0 \omega_d Sin(\omega_d t + \varphi_m)]\,x'(t) \\ &+ [m + \mu_0 Cos(\omega_d t + \varphi_m)]x''(t) \end{aligned} \qquad (7)$$

This non-linear differential equation has no analytic solutions and must be solved numerically. Fig. 2 shows the results of such numerical solutions of Eqn. (7), assuming that $\mathbf{F_d/m}$=0.01, **b/m**=0.01, and $\boldsymbol{\mu_0}$**/m**=0.001. The latter assumption represents only about 10% of the predicted 10 mg mass variation.

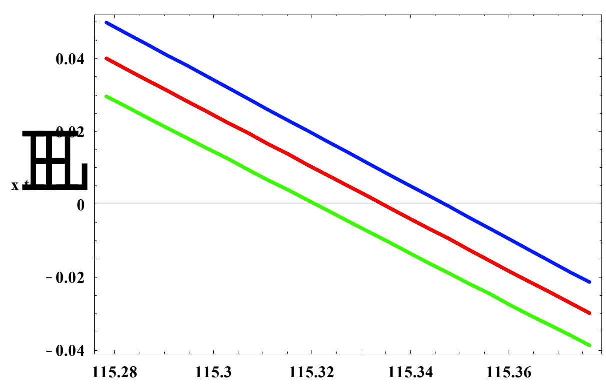

Figure 2. Response phase shifts of the system to variable mass. The central line is the system response with μ_0=0. The other two lines represent μ_0=0.001 with ϕ_m= $-\pi/2$ (low), and ϕ_m=$+\pi/2$ (high). The phase shifts shown are about ±0.04 radians=±2.3 degrees.

We find that when the mass variation has a relative phase of $\pm\pi/2$ with respect to the driving force, it causes a positive or negative phase shift in the response motion by shifts, using the values listed above, of several degrees. Other phases near 0 or π can cause an increase or decrease in the amplitude of oscillation. The experiment we have constructed is designed in an attempt to observe these phase-shift effects.

Experimental Apparatus

Fig. 2 below shows a top view of the mechanical oscillator arrangement, which we call the "Mach Guitar". The barium titanate capacitor test mass is suspended between pairs of tensioned wires, with the tension adjusted so that the resonant vibration frequency for vertical oscillations is about 1-2 kHz.

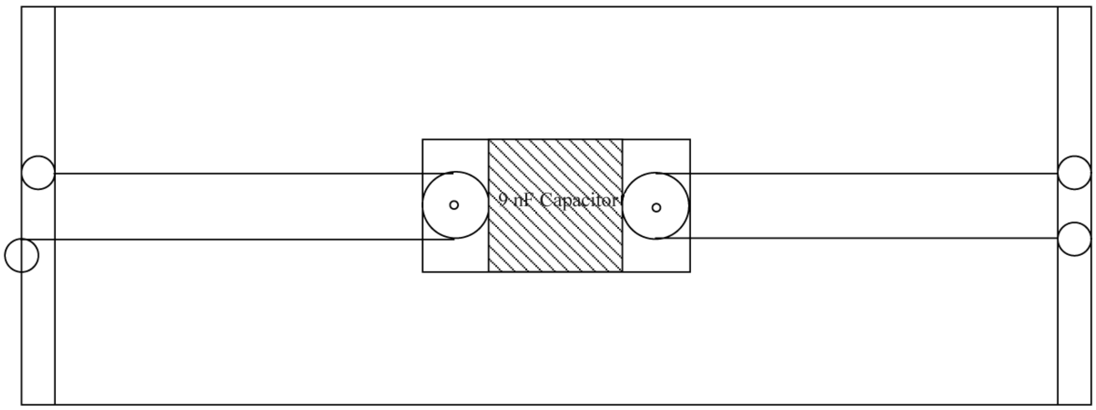

Figure 2 Top view of "Mach Guitar" arrangement. The capacitor is suspended between pairs of tensioned wires that provide the restoring force for the mechanical oscillator. Capacitor drive voltage is supplied through the wires.

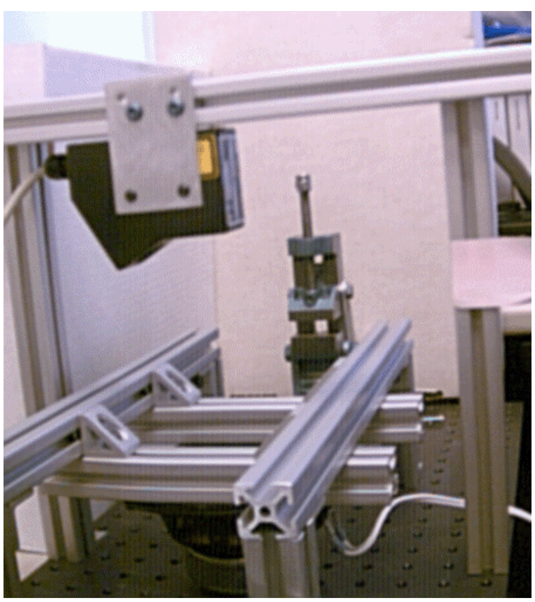

Figure 3 Laser-optics table with oscillator removed, showing voice-coil drive (below) and laser position monitor (above).

Electrical connections for the capacitor drive voltage are supplied through the tensioned wire pairs. The capacitor and its support structure have a net mass of about 1 g.

The restoring force provided by the tensioned wire pairs is $\mathbf{F=-(8T/L)x}$, where $\mathbf{T}$ is the tension in a given wire, $\mathbf{L}$ is the overall length, from bridge to bridge, of the system, and $\mathbf{x}$ is the vertical displacement of the capacitor. Therefore, neglecting the mass of the wire, the resonant frequency of the oscillator is $\boldsymbol{\omega_0=(8T/mL)^{1/2}}$. If $\mathbf{m}$=1 g, $\mathbf{L}$=0.5 m and $\boldsymbol{\omega_0}=2\pi \times 1000$ Hz, then the required tension is 553 lb. This tension can be reached with 13 gauge steel wire.

Fig. 3 shows a view of the laser-optics table (with oscillator removed) that is the foundation of the experiment. A pre-drilled aluminum laser-optics base plate supports the general-purpose aluminum beam structures, on which are mounted (below) the voice-coil drive (a modified audio speaker) for the mechanical oscillator, and (above) the laser position-measuring device shown in Fig. 4.

The Mach Guitar is mounted on the laser-optics base, which provides "bridges" to support for the wires and their tensioning mechanism. Electrical connections to the capacitance are made through the support wires. Below the oscillator is an audio speaker, which drives the oscillator through a light spring. Above the oscillator is a commercial laser position detector, which measures the vertical position of the capacitor's upper surface by electronic triangulation. The laser position sensor is shown in Fig. 4

Fig. 4 Laser position measurement device.

The mass-varying object used in the measurements is a low-loss and low-mechanical-movement barium titanate capacitor with a capacitance of about 9 nF and a voltage rating of 3 kV. This oscillator mass is suspended between pairs of 13 gauge steel wires (0.25 m long on each side) that have been tensioned to about 500 lb to provide a system resonant frequency of about 1000 Hz.

Fig. 5 Trek Model PO923A HV Power Amplifier, used for driving the capacitor at 2 kV and 400 mA.

As previously mentioned, the most challenging problem presented by the present experiment is driving the capacitor to high voltages at audio frequencies. The reason is that all high-voltage amplifiers driving capacitive loads are severely limited by the charging current that they must deliver. We have selected a Trek Model PO923A High Voltage Power Amplifier, shown in Fig. 5, as the capacitor driver. It can drive at voltages up to 2 kV with a peak charging current of up to 400 mA.

The Mach's Principle test employs a Pentium-2 850 MHz computer system with a Windows 98 operating system for experiment control, using control software based on LabView. It consists .of controls for the mechanical oscillator driver and the capacitance driver, a data collection system that records the drive signal and the position measurements, a data recording and retrieval system, and analysis software for processing the data and extracting the phase information of interest.

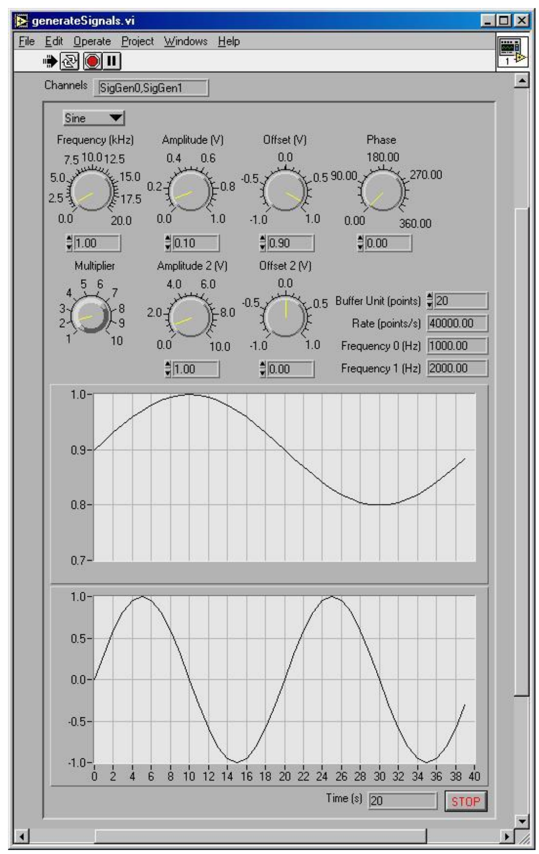

Figure 6. LabView control panel and display for system drivers.

The LabView control panel and display for the experiment is shown in Fig. 6 above. The system generates sine waves with adjustable phases and amplitudes at two frequencies, normally set to differ by a factor of two. The low frequency signal provides the input to the high voltage amplifier that drives the capacitor. The high frequency provides input to an audio amplifier connected to a voice coil that drives the mechanical oscillator.

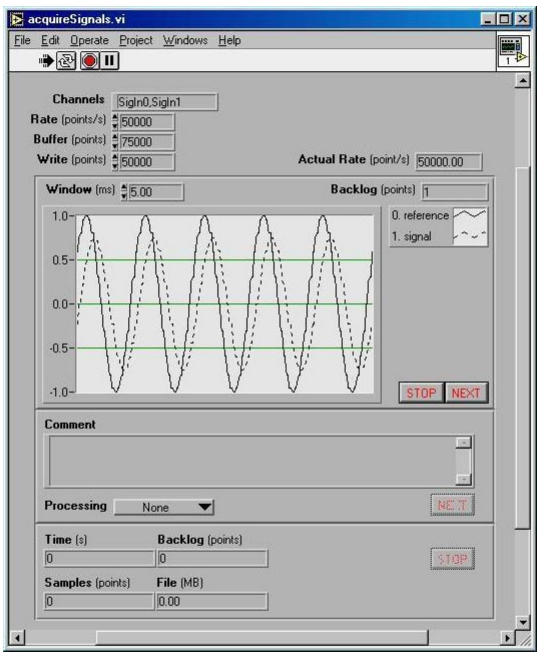

Figure 7. LabView data acquisition and display.

Figure 7 shows the LabView panel for the data collection system. The system samples the mechanical drive voltage and the capacitor position measurement of the mechanical oscillator as separate data streams. These are sampled for real-time display and also recorded on the system hard disk. These data streams can be read back and re-analyzed. The data are analyzed by integration over a long time period to extract the relative phase of the drive and response signals for a given setting of the capacitor drive phase with respect to the mechanical driver.

The processed quantity that will be accumulated in the analysis is the cosine of the relative phase between the driving signal **D(t)** and the mechanical response signal **R(t)**. There are a variety of ways of extracting this signal, but the one we will use initially is:

$$Cos(\varphi) = \frac{\int_0^T [D(t)+R(t)]^2 dt - \int_0^T [D(t)-R(t)]^2 dt}{4\sqrt{\int_0^T D(t)^2 dt \times \int_0^T R(t)^2 dt}} \qquad (8)$$

Here **T** is an arbitrary integration time that increases as data is collected and the running integrals are accumulated. The values of **Cos(ϕ)**, which is near 0 because **ϕ** is approximately **π/2** on resonance, will be compared for the two most extreme settings of the phase of the capacitor drive, which should produce phase shifts like those shown in Fig. 2. We estimate that with a data collection cycle of a few hours, **Cos(ϕ)** can be determined to an accuracy of a few parts in 10^5. This should enable us to determine the shift in **ϕ** to similar accuracy, providing a fairly stringent test of the presence or absence of the Woodward mass variation.

Experiment Status

The experiment is presently being reconfigured on the laser-optic table. The initial cantilever arrangement is being replaced with the new “Mach guitar” mechanical oscillator system described above.

The previous mechanical oscillator, which used a capacitor mass suspended at the free end of an aluminum cantilever, was tested and found to present three serious problems for the experiment: (1) its resonant frequency was fixed by the length and mass of the cantilever and was not easily adjustable, (2) The cantilever mass dominated that of the capacitor, greatly reducing the magnitude of the predicted effect and (3) it was not capable of achieving resonant frequencies above a few hundred Hz. Since the size of the predicted mass-variation effect increases as $\boldsymbol{\omega_0^2}$, this was a serious limitation. However, initial experience with this cantilever oscillator provided valuable experience in operating and testing the position measuring device and the data collection system.

The new Mach Guitar oscillator provides significant improvements over the cantilever in reduced mass and increased operating frequencies, and it offers the additional advantage that it is easily tunable through simple adjustments of the wire tension.

We expect to begin data collection with the new configuration in the next few weeks.

Conclusion

The test of Mach's principle and the Woodward effect described above is not yet completed, but it shows promise of providing an independent test of the predictions that does not depend on the possibility of a reactionless force. The experiment in the present configuration is not as sensitive as the torsion-balance measurements recently reported by Woodward [7]. However, since it is not based on a reactionless force, it may not need that sensitivity.

If the Woodward Effect is observed, it will have important implications for general relativity and cosmology, for validating Mach's Principle, for control of inertia, and possibly for propulsion. If the Woodward Effect is not observed at the sensitivity limit of the experiment, this will also be worth knowing.

Acknowledgements

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